Int. J. Solids Structures Vol. 19, No. 9, pp. 839–841, 1983
Printed in Great Britain

0020-7683/83 $3.00 + .00
Pergamon Press Ltd.

A NOTE ON THE DYNAMIC STRESS FIELD NEAR A PROPAGATING CRACK

K. RAVI-CHANDAR
California Institute of Technology, Pasadena, CA 91125, U.S.A.

(*Received 22 June 1982; in revised form 19 November 1982*)

Abstract – A simple calculation based on the asymptotic crack tip stress distribution is used to determine the stress history due to a propagating crack at points away from the prospective crack line and the results are compared with the experimental measurements in [1]. Very good qualitative correlation is obtained.

In a recent paper entitled *Brittle fracture of plates in tension. Stress field near the crack*, Kinra and Bowers[1] measured the dynamic stress field ahead of a running crack using strain gauges. They calculated the stress history ahead of the crack on the prospective crack line from elastodynamic theory and obtained excellent agreement with the experimental results. While they also obtained very good experimental data for the cases when the strain gauges were mounted away from the crack line, the experiments were not compared with the theory due to the lack of analytical results for these cases. The authors of that paper present the results for comparison with theory if and when it became available. Further, the authors seem to be troubled with the existence of the double peak in the crack-parallel stress history and are careful to point out that this was not due to reflected waves interacting with the crack tip stress field, which arrive a few microseconds after the second peak in the stress history. In this note a very simple calculation, based on the well known asymptotic dynamic stress field at the tip of a moving crack, is suggested to determine the qualitative variation of the dynamic stress field.

Consider a polar coordinate system centered on the crack tip that is moving with a constant velocity, ν, as shown in Fig. 1. We are interested in computing the stresses $\sigma_{\alpha\beta}(t)$ at the point P as the crack propagates and consider the asymptotic stress field given by

$$\sigma_{\alpha\beta}(t) = \frac{K(t)}{\sqrt{(2\pi r)}} f_{\alpha\beta}(\vartheta, \nu): \quad (\alpha, \beta = 1, 2). \tag{1}$$

The angular variation $f_{\alpha\beta}$ is well documented in the literature[2] and is not repeated here. As the crack propagates the (r, ϑ) coordinates of the location P change as a function of time and are easily calculated from the geometry shown in Fig. 1 to be

$$r(t) = [r_0^2 + \nu^2 t^2 - 2r_0 \nu t \cos(\vartheta_0)], \tag{2}$$

$$\vartheta(t) = \arcsin\left[\frac{r_0 \sin\vartheta_0}{r(t)}\right]. \tag{3}$$

If the variation of the stress intensity factor K with time and the velocity of crack propagation are known, eqns (1)–(3) could be used to calculate $\sigma_{\alpha\beta}(t)$. $K(t)$ will in general depend on the specimen geometry and loading conditions but can be written as follows:

$$K(t) = \sigma_0 K_0(g, t, \nu), \tag{4}$$

where σ_0 is the applied far field load in the X_2 direction, and K_0 is a factor that depends on the geometry g, time t and crack velocity ν and has dimensions of *length*$^{1/2}$. From eqns (1) and (4), we have

$$\frac{\sigma_{\alpha\beta}}{\sigma_0} = \frac{f_{\alpha\beta}(\vartheta, \nu)}{\sqrt{(2\pi r)}} K_0(g, t, \nu). \tag{5}$$

In the absence of reflected waves arriving at the crack tip, a situation that is represented by the

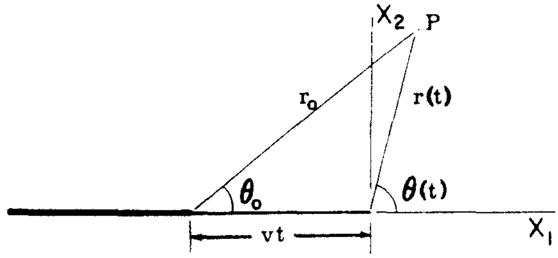

Fig. 1. Crack tip coordinate system.

experimental results in [1], we can consider the time variation of the factor K_0 to be monotonic. Hence the qualitative variation of $\sigma_{\alpha\beta}/\sigma_0$ should be represented by the variation of $f_{\alpha\beta}/\sqrt{(2\pi r)}$. Figures 2 and 3 show the results of the above computations corresponding to two experimental configurations investigated in [1], using the value of crack velocity from their paper. The following points are to be kept in mind in comparing the results of the computation with the experimental results: (a) The calculations have an unknown scaling factor K_0. (b) Since only the singular term in the asymptotic expansion is used, the comparison of magnitudes for large values of r is not meaningful. (c) Steady state propagation is assumed and any transients that may affect the experiment will not be reflected in the calculation. (For example Kinra and Bowers point out the arrival of the plate wave at 10.4 μsec).

For the sake of ready comparison Figs. 4 and 5 from [1] are also reproduced here along with the computed results in Figs. 2 and 3. There exists very good qualitative agreement between the present calculations and the experimental results in [1]. In making the comparison, we note the following points.

(1) As the crack propagates, the radial distance r from the moving crack tip to the location P decreases to a minimum and then increases.The peak in the σ_{22}/σ_0 plot is attained when r reaches a minimum. For the two cases illustrated, the minimum values of r are 7.14 and 19 mm. The ratio of *r/crack length* are 0.094 and 0.25 respectively and at these distances, one should hardly expect the singular term to give precise (quantitative) estimates. If one considers the 0(1) terms, one finds that σ_{12} and σ_{22} are not affected and σ_{11} has a constant (negative) component added to it which already brings the stress history into closer agreement with the experimentally observed history.

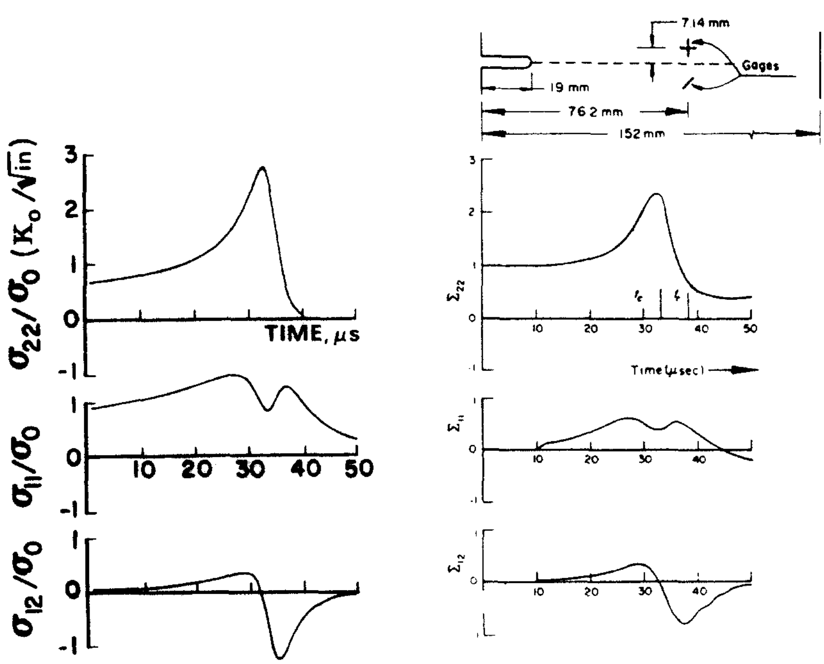

Fig. 2. (a) Calculated stress history and (b) the experimental results of [1].

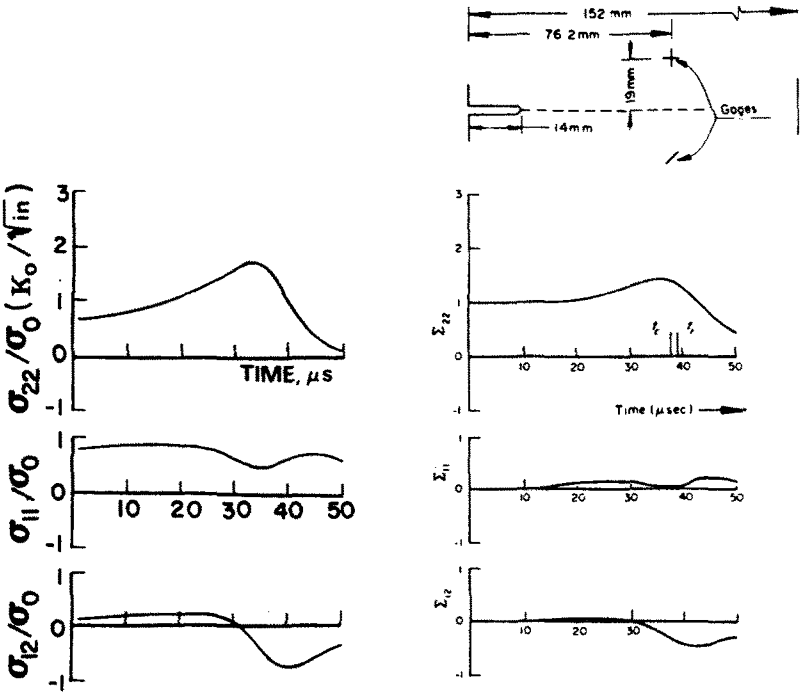

Fig. 3. (a) Calculated stress history and (b) the experimental results of [1].

(2) The σ_{11} stress history exhibits a double peak exactly as the experimental trace indicates. The origin of this double peak is not difficult to explain. The angular variation of f_{11} for a fixed velocity has a double peak[3]. Since the ϑ coordinate for the strain gauge varies from its initial angle ϑ_0 to 180 degrees as the crack passes through, the same double peak is seen in the σ_{11} history. The magnitude varies in the latter case due to the fact that we are plotting $f_{11}/\sqrt{(2\pi r)}$.

(3) Kinra and Bowers indicate that "the pulse shapes are remarkably different for the two cases," but noting that the variation is merely due to the differences in $r(t)$ and $\vartheta(t)$, we believe that the pulses are similar in the two cases.

In conclusion, it is seen that a calculation based on the asymptotic stress field at the tip of a moving crack does well to predict the nature of variation in the stresses.

Acknowledgement—This work was performed under a program entitled "Fundamental Experiments in Dynamic Fracture," sponsored by the National Science Foundation.

REFERENCES

1. V. K. Kinra and C. L. Bowers. *Int. J. Solids Structures* **17**, 175 (1981).
2. L. B. Freund. *Mechanics Today*, (Edited by Nemat-Nasser), Vol. 3, p. 55. Pergamon Press, Oxford (1976).
3. See, e.g., B. R. Lawn and T. R. Wilshaw, *Fracture in Brittle Solids*, p. 55. Cambridge University Press (1975).